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TESTING THE NULL HYPOTHESIS OF STATIONARITY IN FRACTIONALLY INTEGRATED MODELS

Renaud Lacroix

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Testing the null hypothesis of stationarity in fractionally integrated models

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ABSTRACT: In this paper, we show how to estimate consistently the degree of fractional integration at a given frequency θ, for both stationary and non stationary long-memory process. The statistics used are the periodogram for values θ_n which converge to θ with an appropriate rate. We also introduce tests of the hypothesis of stationarity for such processes.

RESUME: L'article propose un estimateur convergent du degré d'integration fraction-

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naire à une fréquence donnée θ pour un processus à mémoire longue stationnaire ou non stationnaire. Les statistiques utilisées sont construites à partir du périodogramme considéré en une fréquence θ_n convergent vers θ avec une vitesse convenable. On en déduit également un test de stationnarité pour de tels processus.

1 Introduction

Let $(X_t)_{t\in\mathbb{Z}}$ be an univariate stationary process with spectral density function $f(\omega)$. The first goal of this paper is to provide a test of the hypothesis $f(\theta) = 0$ when f is not supposed to be a globally regular function, that is, f is not continuously differentiable[1] on $[-\pi, \pi]$. This situation occurs in particular when X_t is obtained by seasonally differencing at frequency θ a process admitting long memory dynamic at some frequency $\theta^{'}$, with eventually $\theta^{'} = \theta$. Recently, Hidalgo (1996) showed that classical kernel estimators of f are still consistent for each non singular value of the spectrum of a long memory stationary process. However, these results can not be used for the test $f(\theta) = 0$, because, similarly to the regular case, the asymptotic variance of the estimator is proportional to $f^2(\theta)$. In the regular case, locally optimal parametric tests have been proposed by (Tanaka (1996), Tam and Reinsel (1997)). Lacroix (1999) discussed also non-parametric tests.

For some fixed frequency θ in $[0, \pi]$, this paper presents a test of the hypothesis $f(\theta) = 0$ bases on the behavior of the periodogramme for points "near" θ. This approach is rather classical in non-parametric spectral methods. For instance, Lobato and Robinson (1998) have recently proposed a test of the stationary hypothesis against fractionally alternatives. Their test uses frequencies taken from an asymptotic degenerate segment around $\theta = 0$. Our approach exploits the following well-known fact: *if* $f(\theta) = 0$, *then the periodogram is a consistent estimator of* $f(\theta)$ (see eg

[1] We will say that X_t is not regular, not to be confused with properties of the Wold representation.

Priesley (1988)). We demonstrate in this paper that this property still holds in a neighborhood of θ, which allow us to build test statistics which are insensitive to irregularity (such as long-memory behavior) at some other frequencies in the spectrum, a property shared by non-parametric estimators developed in the recent years.

The paper covers the following grounds. Section 2 outlines the assumptions and derives asymptotic results for the periodogram. Section 3 shows how to apply this result to identify the degree of fractional "over-differencing", a topic which is closely related to the nullity of the spectrum at frequency θ. Section 4 is devoted to a test of stationarity for a process with long memory.

We begin with some notations used throughout the paper.

We observe a finite sample $\{X_k, 1 \leq k \leq n\}$ of a real valued stationary, purely non deterministic process (PND, in short) $(X_t)_{t\in\mathbb{Z}}$ which will be moreover supposed centered. Its Wold expansion is:

$$X_t = \sum_{j=0}^{\infty} \Psi_j \varepsilon_{t-j} \tag{1}$$

$(\varepsilon_t)_{t\in\mathbb{Z}}$ is the innovation of X_t, $\Psi_0 = 1$, $\sum_{j=0}^{\infty} \Psi_j^2 < \infty$. We define the Wold function as the complex function $C_X(z) = \sum_{j=0}^{\infty} \Psi_j z^j$ for all $|z| < 1$. The spectral density of X_t can be expressed as $f(\omega) = \frac{\sigma^2}{2\pi} \left|C_X\left(e^{-i\omega}\right)\right|^2$ with $\sigma^2 = \mathbb{E}\left(\varepsilon_t^2\right)$.

Let $\Delta_n(u) = \sum_{k=0}^{n-1} e^{iku}$ be Dirichlet kernel; it satisfies $\int_{-\pi}^{\pi} \left|\Delta_n(x)\right|^2 dx = n$ and:

$$|x\Delta_n(x)| \leq 2 \text{ pour } 0 \leq |x| < \pi \tag{2}$$

In the sequel, the symbol "$\Rightarrow$" denotes weak convergence when n tends to infinity. Capital letters $C, C', C'', \ldots$ are constant independent of n or $\omega \in [0, \pi]$. These constants will take different values for each new occurrence. We note, for $p \geq 1$, $\|X\|_p = (\mathbb{E}|X|^p)^{1/p}$ for any complex random variable X admitting moments up to order p. Finally, we denote the periodogram of any sequence $(Z_t)_{1\leq k\leq n}$ by:

$$\mathbb{I}_Z(\omega) = \frac{1}{n}\left|\sum_{k=1}^{n} e^{-ik\omega} Z_k\right|^2, \text{ for all } \omega$$

2 Local behavior of the periodogram

We suppose that $(\varepsilon_t)_{t\in\mathbb{Z}}$ is a martingale difference sequence adapted to some filtration $(\mathcal{F}_t)$ and:

$$\mathbf{H}_\varepsilon : \begin{cases} \mathbb{E}(\varepsilon_t^2 \mid \mathcal{F}_{t-1}) = \sigma_t^2,\ \mathbb{E}(\varepsilon_t^2) = \sigma^2 \\ \sigma_t^2 = \sigma^2 + \sum_{k=1}^{\infty} c_k\left(\varepsilon_{t-k}^2 - \sigma^2\right),\ \sum_{k=1}^{\infty}|c_k| < \infty \\ \sum_{k=1}^{\infty} c_k z^k \neq 1 \text{ si } |z| \leq 1 \\ \sup_t \mathbb{E}\left(\varepsilon_t^4\right) < \infty \end{cases}$$

These hypotheses allow for ARCH or GARCH dynamic for ε_t.

We need the following theorem which is proved in Lacroix (1999).

Theorem 1 *Let $\theta_n \in]0,\pi[$ be a sequence converging to θ, and such as $n(\theta_n - \theta) \to -\infty$ if $\theta = \pi$, $n\theta_n \to +\infty$ if $\theta = 0$. Then:*

$$\mathbb{I}_\varepsilon(\theta_n) \Rightarrow \frac{\sigma^2}{2}\chi^2(2) \tag{3}$$

Remark 1 *If we consider two distinct frequencies θ_1 and $\theta_2 \in [0,\pi]$, and two sequences $\theta_{1,n}$ and $\theta_{2,n}$ satisfying the assumptions of the theorem then we have the* *stronger result:*

$$\begin{pmatrix} \mathbb{I}_{\varepsilon}(\theta_{n,1}) \\ \\ \mathbb{I}_{\varepsilon}(\theta_{n,2}) \end{pmatrix} \Rightarrow \frac{\sigma^2}{2} \begin{pmatrix} \chi_1^2(2) \\ \\ \chi_2^2(2) \end{pmatrix} \tag{4}$$

$\chi_1^2(2)$ *and* $\chi_2^2(2)$ *are* *independent. This explains why it is possible to consider several frequencies* θ_j*, and to treat them separately from each other.*

We describe now the assumptions relative to the smoothness of the spectral density.

$$\mathbf{H}_f : \begin{cases} \mathbf{H}_1 : \exists\ V(\theta) \text{ neighborhood of } \theta \text{ such as } \Psi \text{ is continuous on } V(\theta) \\ \text{and differentiable on } V^*(\theta) = V(\theta) \backslash \{\theta\} \\ \mathbf{H}_2 : \exists C_1 > 0,\ \delta \in [1,2],\ |\Psi(\omega)| \geq C_1 |\omega - \theta|^{\delta} \text{ for } \omega \in V(\theta) \\ \mathbf{H}_3 : \exists C_2 > 0,\ d \in [1-2\delta, 1],\ |\Psi'(\omega)| \leq C_2 |\omega - \theta|^{-d} \text{ for } \omega \in V^*(\theta) \end{cases}$$

Note that these assumptions are elative to local behavior of the spectrum. They do not exclude that f diverges at some point $\theta_1 \neq \theta$, which is typically the case of long memory at θ_1. f may, or may not be differentiable at θ, depending upon the values of δ and d. The hypothesis $\mathbf{H}_2$ implies that Ψ has only one (possible) zero at θ on $V(\theta)$. From $\mathbf{H}_1$, this hypothesis is trivially fulfilled with $\delta = 1$ if $\Psi(\theta) \neq 0$. $\mathbf{H}_3$ is satisfied in particular with $\Psi(\omega) = \left(1 - e^{i(\omega-\theta)}\right)^a$, $a \in \left]0,1\right[$ and $d = 1 - a$ and $\delta = 1$. If Ψ is continuously differentiable in a neigborhood of θ, one can choose $d = 0$.

We remark now that f is also continuous on $V(\theta)$, and differentiable on $V^*(\theta)$ with $f(\omega) \geq C_1' |\omega - \theta|^{2\delta}$ for $\omega \in V(\theta)$. Next, $f'(\omega) = C^{ste} \times \mathrm{Re}\left[\Psi'(\omega)\overline{\Psi(\omega)}\right]$ so $f'(\omega)$ satisfies $|f'(\omega)| \leq C_2' |\omega - \theta|^{-d}$ because Ψ is bounded.

We precise now the behavior of f around θ when this function vanishes at this point. Take $\varepsilon > 0$ such as $[\theta - \varepsilon, \theta + \varepsilon] \subset V(\theta)$ and $\left(\omega, \widetilde{\theta}\right)$ such as $\theta + \varepsilon > \omega > \widetilde{\theta} > \theta$.
$\mathbf{H}_3$ implies $f(\omega) - f\left(\widetilde{\theta}\right) = \int_{\widetilde{\theta}}^{\omega} f'(u)\, du$ hence
$\left|f(\omega) - f\left(\widetilde{\theta}\right)\right| \leq \frac{C_2}{1-d}\left[|\omega - \theta|^{1-d} - \left|\widetilde{\theta} - \theta\right|^{1-d}\right]$. Let $\widetilde{\theta}$ goes to θ ; by continuity of f, and $d \leq 1$ $|f(\omega) - f(\theta)| \leq \frac{C_2}{1-d}|\omega - \theta|^{1-d}$. Hence under $H_0 : f(\theta) = 0$, we get:

$$\forall \omega \in V(\theta), C_1^{'}|\omega - \theta|^{2\delta} \leq f(\omega) \leq \frac{C_2}{1-d}|\omega - \theta|^{1-d} \tag{5}$$

It is clear that if $\delta > 1$ and $d < 0$, then f is differentiable at θ.

We can now state the:

Lemma 2 *Under the hypothesis* $\mathbf{H}_1$ *and* $\mathbf{H}_3$*, and if* $\theta_n = \theta + \lambda_n$ *with* $\lambda_n \to 0$*:*

$$\mathbb{E}\left|\mathbb{I}_X(\theta_n) - \mathbb{I}_\varepsilon(\theta_n) f(\theta_n)\right| = O\left(\sqrt{\frac{1 + \log(n|\lambda_n|)}{n|\lambda_n|}}\right)$$

Proof: see the appendix.

■

When f is continuously differentiable on the real line, one has the classical bound (see Priestley (1988)):

$$\mathbb{E}\left|\mathbb{I}_X(\theta_n) - \mathbb{I}_\varepsilon(\theta_n) f(\theta_n)\right| = O\left(\frac{1}{\sqrt{n}}\right)$$

If we compare this result to (5), it is seen that the term $O\left(\frac{1}{\sqrt{n}}\right)$ has been replaced by the less precise bound $O\left(\sqrt{\frac{1+\log(n|\lambda_n|)}{n|\lambda_n|}}\right)$. This is of course the price to pay when working with minimal assumptions about the Wold function.

Theorem 3 *Under the hypothesis* $\mathbf{H}_f$ *and* $\mathbf{H}_\varepsilon$*, if* $\frac{1}{n|\lambda_n|^5} + \frac{\log(n|\lambda_n|)}{n|\lambda_n|^5} = o(1)$ *for* $\delta = 1$*, and* $\frac{1}{n|\lambda_n|^9} + \frac{\log(n|\lambda_n|)}{n|\lambda_n|^9} = o(1)$ *for* $\delta \in]1, 2]$*:*

$$\frac{\mathbb{I}_X(\theta_n)}{f(\theta_n)} \Rightarrow \chi^2(2) \tag{6}$$

Proof: $\mathbb{E}\left|\frac{\mathbb{I}_X(\theta_n)}{f(\theta_n)} - \mathbb{I}_\varepsilon(\theta_n)\right| = O\left(\sqrt{\frac{a_n}{\lambda_n^{4\delta}}}\right) = O\left(\sqrt{\frac{1+\log(n\lambda_n)}{n\lambda_n^{4\delta+1}}}\right)$ with $\mathbf{H}_2$. Hence, $\frac{\mathbb{I}_X(\theta_n)}{f(\theta_n)} - \mathbb{I}_\varepsilon(\theta_n) = o_p(1)$. An application of lemma 2 yields the desired result.

■

Note that even if $f(\theta) = 0$, f is not necessarily symmetric around θ, a situation which is different from the case in which f is twice differentiable at θ. It means that in finite samples, the values of $\frac{\mathbb{I}_X(\theta_n)}{f(\theta_n)}$ may be sensitive to the choice of the sequence λ_n, and more precisely to the sign of λ_n. Conversely, under the alternative hypothesis, $f(\theta) > 0$, and f is generally always asymmetric, whenever it is smooth or not.

Remark 2 *If* $f(\theta_n) = 0$ *and* $\frac{1+\log(n\lambda_n)}{n\lambda_n^2} = o(1)$*, from (5) we get:* $\mathbb{E}|\mathbb{I}_X(\theta_n) - f(\theta_n)\mathbb{I}_\varepsilon(\theta_n)| = O(\sqrt{a_n}) \times O\left(\sqrt{f(\theta_n) + a_n}\right) = O\left(\sqrt{a_n|\lambda_n|^{1-d}}\right)$*. It yields* $\frac{\mathbb{I}_X(\theta_n)}{f(\theta_n)} - \mathbb{I}_\varepsilon(\theta_n) = o_p\left(\lambda_n^{(1-d)/2}\right)$*: the normalized periodogram converges faster towards its limit when the spectral density vanishes.*

3 Identification of a fractional process

We give now an application of this result to a class of processes which admit a spectral density $f(\omega)$ such as, in the neighborhood of θ:

$$f(\omega) \sim C(\omega - \theta)^{2d} \text{ for some } d \in [0, 2] \tag{7}$$

The spectrum does not vanish like a polynomial when $d \in \left]0,1\right[$. Robinson (1995) showed how to get an asymptotically normal estimator for parameters C and d when $\theta = 0$ et $d \in \left]-1/2\,,1/2\right[$, based on the maximization of a "pseudo-likelihood". This criterium is then discretized and calculated from the values of the periodogram at frequencies $\theta_{j,n} = \frac{2\pi j}{n}$ for $1 < j < m$ with m tending to infinity with $mn^{-1} \to 0$. These results have been extended to the case $\theta \in \left]0,\pi\right[$ by Arteche (1998). Note that for $d \in \left]0,1/2\right[$, f vanishes at θ. Robinson's statistic may be used to test the hypothesis $f(\theta) = 0$. We propose now to test the hypothesis $f(\theta) = 0$ valid for $d \geq \frac{1}{2}$ as an application of the theorem proved above.

We start with the following leading example:

$$\begin{aligned} &X_t = \left(1 - 2\cos\theta_1 B + B^2\right)^{d_1}\left(1 - 2\cos\theta_2 B + B^2\right)^{d_2} Z_t \\ &d_1, d_2 \in \left[0,2\right[;\theta_1,\theta_2 \in \left]0,\pi\right], \theta_1 \neq \theta_2 \end{aligned} \tag{8}$$

Z_t is a second order stationary process with spectral density $f_Z(\omega)$ continuously differentiable in the neighborhood of θ_1 and strictly positive at this point. In this example, d_2 and θ_2 are unknown nuisance parameters. We assume that ε_t, innovation of Z_t, satisfies the hypothesis of theorem 2.

We recall that (see Gray et al. (1989)) for $d \in \mathbf{R}\diagdown\mathbf{Z}$, and $|\eta| \leq 1$, Gegenbauer functions are defined by the expression:

$$\begin{aligned} &g(d,z) = \left(1 - 2\eta z + z^2\right)^d = \textstyle\sum_{j=0}^{\infty} c_j(d,\eta)\, z^j \text{ for } |z| < 1 \\ &\text{with } c_j(d,\eta) = \textstyle\sum_{k=0}^{\left[\frac{d}{2}\right]} \frac{(-1)^k \Gamma(-d+j-k)(2\eta)^k}{\Gamma(-d)\Gamma(k+1)\Gamma(j-2k+1)} \end{aligned} \tag{9}$$

If $\theta \neq \pi$, $c_k(d,\cos\theta) \sim_{k\to\infty} \frac{\cos\left[(k-d)\theta + d\frac{\pi}{2}\right]}{\Gamma(-d)\sin^{-d}(\theta)}\left(\frac{2}{k}\right)^{1+d}$ so $c_k(d,\cos\theta) = O\left(k^{-d-1}\right)$

If $\theta = \pi$, $c_j(d,-1) = (-1)^j \frac{\Gamma(j-2d)}{j!\Gamma(-2d)} \sim_{j\to\infty} \frac{j^{-2d-1}}{\Gamma(-2d)} = O\left(j^{-2d-1}\right)$

When $d \in \mathbf{N}$, $g(d,z)$ is polynomial. In this case $c_k(d, \cos\theta)$ is zero for $k \geq 2d+1$.

For $d_1, d_2 \in [0,+\infty[$ the product operator $g(d_1, B)\, g(d_2, B)$ is absolutely summable, and X_t is defined as the limit with probability one of an absolutely convergent series. Next, X_t stationary, with spectral density:

$$f(\omega) = 4(\cos\omega - \cos\theta_1)^{2d_1} \times 4(\cos\omega - \cos\theta_2)^{2d_2} \times f_Z(\omega) \tag{10}$$

We suppose θ_1 is known, we want to test the hypothesis $f(\theta_1) = 0$ against $f(\theta_1) \neq 0$, which takes the form:

$$\begin{aligned} \mathbf{H}_0 &: \; d_1 \in]0,2] \\ \mathbf{H}_a &: \; d_1 = 0 \end{aligned} \tag{11}$$

When the dynamic of Z_t is completely parametrized (via some ARMA specification for instance), under the normality assumption for ε_t, and the restriction $d_1, d_2 \in]0,1[$, one may estimate the parameters (θ_2, d_1, d_2) with maximum likelihood. But it appears to be a difficult task from a numerical point of view, and moreover, the case $d_1 \geq 1$ is not covered. Observe now that in a neighborhood of θ_1, $f(\omega) \sim C(\omega - \theta_1)^{2d_1} f_Z(\theta_1) \sim C'(\omega - \theta_1)^{2d_1}$ if $\theta_1 \neq \pi$, and $f(\omega) \sim C'(\omega - \theta_1)^{4d_1}$ if $\theta_1 = \pi$ (in this case $\mathbf{H}_0$ must be replaced by $d_1 \in]0,1[$). The hypothesis $\mathbf{H}_f$ are fulfilled, with $\delta = \max(1, d_1)$, $d = 1 - 2d_1$ for $\theta_1 \neq \pi$ and $d = 1 - 4d_1$ for $\theta_1 = \pi$. For $\theta_1 \neq \pi$ it yields, for a sequence θ_n verifying the assumption of theorem 3:

$$\frac{\mathbb{I}_X(\theta_n)}{C'(\theta_n - \theta_1)^{2d_1}} \Rightarrow \chi^2(2) \tag{12}$$

This result depends upon the nuisance parameter:

$$C^{'} = 16\left(\sin\theta_1\right)^{2d_1} \times \left(\cos\theta_1 - \cos\theta_2\right)^{2d_2} \times f_Z(\theta_1)$$

However we have the following result which will allow us to get rid of this parameter.

Theorem 4 *If* $\theta_n^1, \theta_n^2, \theta_n^3, \theta_n^4 \in \left]0, \pi\right[$ *are four sequences converging to* θ_1, $\theta_n^j = \theta_1 + e_j^{-1}(n)$, *such as* $|e_j(n)| \to +\infty$*:*

$e_j(n) = o\left(n^{1/9}\right)$, *and if* $j_1 \neq j_2$ *then* $e_{j_1}(n) \neq e_{j_1}(n)$, $e_{j_1}(n) = o\left[e_{j_2}(n)\right]$ *or* $e_{j_2}(n) = o\left[e_{j_1}(n)\right]$

If $\left(\xi_n^{r,1}, \xi_n^{r,2}\right)^{'} = \left(\left|\frac{e_2(n)}{e_1(n)}\right|^{d_1}\sqrt{\frac{\mathbb{I}_X\left(\theta_n^1\right)}{\mathbb{I}_X\left(\theta_n^2\right)}}, \left|\frac{e_4(n)}{e_3(n)}\right|^{d_1}\sqrt{\frac{\mathbb{I}_X\left(\theta_n^3\right)}{\mathbb{I}_X\left(\theta_n^4\right)}}\right)$*, then:*

$$\left(\xi_n^{r,1}, \xi_n^{r,2}\right)^{'} \Rightarrow \left(\sqrt{F_{2,2}^{(1)}}, \sqrt{F_{2,2}^{(2)}}\right)$$

$$F_{2,2}^{(1)} \text{ is independent of } F_{2,2}^{(2)}$$

$F_{2,2}$ *is the Fisher law with (2,2) degrees of freedom.*

Proof: From theorem 19 of Lacroix (1999), we know that:

$$\begin{pmatrix} \mathbb{I}_\varepsilon\left(\theta_{n,1}\right) \\ \mathbb{I}_\varepsilon\left(\theta_{n,2}\right) \\ \mathbb{I}_\varepsilon\left(\theta_{n,3}\right) \\ \mathbb{I}_\varepsilon\left(\theta_{n,4}\right) \end{pmatrix} \Rightarrow \frac{\sigma^2}{2}\begin{pmatrix} \chi_1^2(2) \\ \chi_2^2(2) \\ \chi_3^2(2) \\ \chi_4^2(2) \end{pmatrix}$$

The $\chi^2(2)$ are independent from each other. Now, lemma 2 yields:

$$\begin{pmatrix} \mathbb{I}_X\left(\theta_{n,1}\right) f\left(\theta_{n,1}\right)^{-1} \\ \mathbb{I}_X\left(\theta_{n,2}\right) f\left(\theta_{n,2}\right)^{-1} \\ \mathbb{I}_X\left(\theta_{n,3}\right) f\left(\theta_{n,3}\right)^{-1} \\ \mathbb{I}_X\left(\theta_{n,4}\right) f\left(\theta_{n,4}\right)^{-1} \end{pmatrix} \Rightarrow \frac{\sigma^2}{2}\begin{pmatrix} \chi_1^2(2) \\ \chi_2^2(2) \\ \chi_3^2(2) \\ \chi_4^2(2) \end{pmatrix}$$

We apply now the continuous mapping theorem, and use the property:

$$f\left(\theta_{n,i}\right)^{-1} f\left(\theta_{n,j}\right)^{-1} \sim \left(\frac{e_i(n)}{e_j(n)}\right)^{2d_1} \quad \text{when } n \to +\infty$$

■

In the sequel we take $e_2(n) = n^{\alpha}$ ($\alpha < \frac{1}{9}$), $e_1(n) = \log(n)$, $e_4(n) = n^{\alpha'}$ ($\alpha' < \frac{1}{9}$ et $\alpha' \neq \alpha$), $e_3(n) = (\log(n))^{\gamma}$, $\gamma > 0$. From the continuous mapping theorem we get:

$$d_1 \log\left(\frac{n^{\alpha}}{\log(n)}\right) + \log\left\{\sqrt{\frac{\mathbb{I}_X\left(\theta_n^1\right)}{\mathbb{I}_X\left(\theta_n^2\right)}}\right\} \Rightarrow \frac{1}{2}\log F_{2,2}^{(1)} \equiv Y^{(1)}$$

with $Y^{(1)}$ logistic law, with c.d.f $F_Y(x) = \frac{1}{1+\exp(-2x)}$

Define now $\widehat{d_1} = -\frac{\log S_n^{(1)}}{\log\left(\frac{n^{\alpha}}{\log(n)}\right)}$, $S_n^{(1)}$ being the term within brackets in the last expression above.

$$\log\left(\frac{n^{\alpha}}{\log(n)}\right) \times \left[\widehat{d_1} - d_1\right] \Rightarrow Y^{(1)} \tag{13}$$

This result implies that $\widehat{d_1}$ is a consistent estimator of d_1, with a very slow rate of convergence, namely $O\left(\log^{-1} n\right)$. (13) can also be written as:

$$\left(\frac{n^{\alpha}}{\log(n)}\right)^{\left[\widehat{d_1}-d_1\right]} \Rightarrow \exp\left[Y^{(1)}\right]$$

Then, $\left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right)^{\widehat{d_1}-d_1} = \exp\left[\left(\widehat{d_1} - d_1\right)\log\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right]$
But $\log\left(\frac{n^{\alpha}}{\log(n)}\right) \sim \frac{\alpha}{\alpha'}\log\left(\frac{n^{\alpha'}}{(\log(n))^{\gamma}}\right)$

Hence:

$$\left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\frac{C_4}{C_3}\right)^{\widehat{d_1}-d_1} = \exp\left[\left(\widehat{d_1} - d_1\right)\left\{\frac{\alpha'}{\alpha}\log\left(\frac{n^{\alpha}}{\log(n)}\right)(1+o(1))\right\}\right]$$

Now $\left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right)^{\widehat{d_1}-d_1} = \left[\left(\frac{n^{\alpha}}{\log(n)}\right)^{\widehat{d_1}-d_1}\right]^{\frac{\alpha'}{\alpha}} \times \exp\left[\frac{\alpha'}{\alpha}\log\left(\frac{n^{\alpha}}{\log(n)}\right)\left(\widehat{d_1}-d_1\right)\times o(1)\right]$

The exponential is $O_p(1)$, so:

$$\left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right)^{\widehat{d_1}-d_1} \Rightarrow \left(\exp\left[Y^{(1)}\right]\right)^{\frac{\alpha'}{\alpha}}$$

Moreover, $\left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right)^{d_1+2}\sqrt{\frac{\mathbb{I}_X\left(\theta_n^3\right)}{\mathbb{I}_X\left(\theta_n^4\right)}} \Rightarrow \sqrt{F_{2,2}^{(2)}}$, or equivalently:

$$\left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right)^{d_1} S_n^{(2)} \Rightarrow \sqrt{F_{2,2}^{(2)}} = \exp\left[2Y^{(2)}\right]$$

with $S_n^{(2)} = \left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right)^{2}\sqrt{\frac{\mathbb{I}_X\left(\theta_n^3\right)}{\mathbb{I}_X\left(\theta_n^4\right)}}$

From the convergence of the couple $\left(\xi_n^{r,1},\xi_n^{r,2}\right)'$ and the continuous mapping theorem, we get:

$$\begin{pmatrix} \left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right)^{d_1} S_n^{(2)} \\ \left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right)^{\widehat{d_1}-d_1} \end{pmatrix} \Rightarrow \begin{pmatrix} \exp\left[2Y^{(2)}\right] \\ \left(\exp\left[Y^{(1)}\right]\right)^{\frac{\alpha'}{\alpha}} \end{pmatrix} \tag{14}$$

Once more we apply the continuous mapping theorem with the application $(x,y) \rightarrow x \times y$. It yields:

$$\left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right)^{\widehat{d_1}} S_n^{(2)} \Rightarrow \exp\left[2Y^{(2)}\right]\left(\exp\left[Y^{(1)}\right]\right)^{\frac{\alpha'}{\alpha}}$$

This result is valid for all $d_1 \in [0,2[$.

We are now in position to develop a test of $\mathbf{H}_0$ against $\mathbf{H}_a$.

Under $\mathbf{H}_0$, $d_1 > 0$ and:

$$\log n \times S_n^{(2)} = \left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right)^{d_1} S_n^{(2)} \times \frac{(\log(n))^{\gamma(d_1+1)}}{n^{d_1\alpha'}}$$

The first term converges in law, whereas the second converges to zero since $d_1 > 0$.

We get $\log n \times S_n^{(2)} \xrightarrow{P} 0$, then $n^{S_n^{(2)}} \xrightarrow{P} 1$

Now under $\mathbf{H}_a$, $d_1 = 0$ and:

$$S_n^{(2)} \Rightarrow \exp\left[2Y^{(2)}\right]$$

then $\log n \times S_n^{(2)} \xrightarrow{P} +\infty$.

We define an asymptotic consistent test from the statistic:

$$\kappa_n\left(\alpha, \alpha^{'}, \gamma\right) = n^{S_n^{(2)}} \times \left(\frac{n^{\alpha^{'}}}{\log^{\gamma}(n)}\right)^{\widehat{d_1}} \times S_n^{(2)} \tag{15}$$

and its associated critical intervals:

$$W_n = \{\kappa_n > l_{1-a}\}$$

$G\left(l_{1-a}\right) = 1 - \alpha$ where G is the c.d.f.of the variable $\mathbb{L}_{\infty} = \exp\left[2Y^{(2)} + \frac{\alpha^{'}}{\alpha}Y^{(1)}\right]$.

4 A test of stationarity

Let us consider X_t such as:

$$\left(1 - 2\cos\theta B + B^2\right)^d X_t = Z_t \tag{16}$$

Z_t is a process with regular spectral density, and d is a real number.

When $d \in \left]-1/2\,,1/2\right[$ and $\theta \in \left]0, \pi\right[$ or $d \in \left]-1/4\,,1/4\right[$ and $\theta = \pi$, X_t is stationary and invertible. If moreover $d > 0$, its autocorrelations decrease very slowly, with a cyclical pattern of frequency θ: this is the situation of long memory at frequency θ. Such model seem appropriate for the modelling of seasonal patterns (take $\theta = \frac{\pi}{2}$

or π in the case of quarterly data). When $d = 1$, this is the classical situation of non-stationarity, in which past shocks are always persistent: we may say that such processes have infinite memory. Values of d between $1/2$ and 1 lead to a continuum of intermediate descriptions of non-stationarity. When $d > 1$ and approaches 2 from below, the process becomes more and more smooth and close to a model with two unit roots, a situation which occurs frequently with nominal macroeconomic series. In general, descriptive statistics fail to discriminate between these representations. For instance, spectral density estimators will show a peak at frequency θ, whatever the value of $d \in]0, 2]$ and we are unable to make a clear decision about the magnitude of this peak. Robinson (1994) proposed in a semi-parametric framework an asymptotically normal score-test for $\mathbf{H}_0 : d = 1/2$ against $\mathbf{H}_a : 0 < d < 1/2$. Recall that we may also estimate d by CLS when $d \in]-1/2\,, 1/2[$. (Chung (1996)). We propose now a much simpler method to handle the testing-problem. We first differentiate twice the process:

$\widetilde{X_t} = (1 - 2\cos\theta B + B^2)^2 X_t = (1 - 2\cos\theta B + B^2)^{\widetilde{d}} Z_t$, with $\widetilde{d} = 2 - d \in [0, 2]$. $\widetilde{X_t}$ is always stationary, and from the preceding section we can consistently estimate $\widetilde{d}$, and so d. We propose the test:

$$\theta \neq \pi : \quad \begin{array}{l} \mathbf{H}_0 : d \in]0, 1/2[\, : X_t \text{ stationary, long memory} \\ \mathbf{H}_a : d \in [1/2\,, 2] : X_t \text{ non-stationary} \end{array}$$

$$\theta = \pi : \quad \begin{array}{l} \mathbf{H}_0 : d \in]0, 1/4[\\ \mathbf{H}_a : d \in [1/4\,, 1] \end{array}$$

It is equivalent to $\widetilde{\mathbf{H}}_0 : \widetilde{d} \in \left]3/2 , 2\right[$ against $\widetilde{\mathbf{H}}_a : \widetilde{d} \in [0, 3/2]$ for $\theta \neq \pi$, and $\widetilde{\mathbf{H}}_0 : \widetilde{d} \in \left]1/4 , 1/2\right[$ against $\widetilde{\mathbf{H}}_a : \widetilde{d} \in [1/2 , 3/4]$. With the notations of the previous section we obtain from $\widetilde{X_t}$:
$\log n \sqrt{\frac{n^{\alpha'}}{\log^{\gamma}(n)}} \times S_n^{(2)} = \left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right)^{\widetilde{d}} S_n^{(2)} \times \log n \left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right)^{1/2-\widetilde{d}} \xrightarrow{P} 0$ under $\mathbf{H}_0$, and diverges to $+\infty$ under $\mathbf{H}_a$.

One defines a consistent test with the statistic below whose limit law is identical under the null to the limit law of $\kappa_n\left(\alpha, \alpha', \gamma\right)$:

$$\kappa_n'\left(\alpha, \alpha', \gamma\right) = \exp\left(\log n \sqrt{\frac{n^{\alpha'}}{\log^{\gamma}(n)}} \times S_n^{(2)}\right) \times \left(\frac{n^{\alpha'}}{\log^{\gamma}(n)}\right)^{\widehat{d_1}} \times S_n^{(2)} \qquad (17)$$

5 Concluding remarks

We show in this paper how to estimate in a very simple way the memory parameter d for models of both non-stationary and stationary fractionally integrated time series. The main drawback of this approach is that the estimator converges rather slowly (it is indeed only $\log n$-consistent). Some simulations experiment are then needed in order to assess the typical sample size needed for the estimator and its related testing procedures to be meaningful.

6 Appendix: proof of lemma 2

We suppose to simplify the exposition that $\lambda_n > 0$ if $\theta < \pi$, and $\lambda_n < 0$ if $\theta = \pi$. We follow Robinson (1995b), theorem 2. We have to prove that:

$$\begin{aligned} &i)\ \mathbb{E}\mathbb{I}_X(\theta_n) = f(\theta_n) + O(a_n) \\ &ii)\ \mathbb{E}\mathbb{J}_X(\theta_n)\overline{\mathbb{J}_\varepsilon(\theta_n)} = \tfrac{\sigma^2}{2\pi}\Psi(\theta_n) + O(a_n) \\ &\text{with } a_n = \tfrac{1+\log(n\lambda_n)}{n\lambda_n} \end{aligned} \tag{18}$$

Indeed, from relation (3-17) of Robinson (1995b), and $\mathbb{E}\mathbb{I}_\varepsilon(\theta_n) \equiv \frac{\sigma^2}{2\pi}$, the lemma yields:

$$\mathbb{E}\left|\mathbb{I}_X(\theta_n) - f(\theta_n)\mathbb{I}_\varepsilon(\theta_n)\right| = O\left(\sqrt{a_n\left[1+|\Psi(\theta_n)|\right]}\right)\times O\left(\sqrt{f(\theta_n) + a_n\left[1+|\Psi(\theta_n)|\right]}\right) \tag{19}$$

f and Ψ are bounded on $V(\theta)$, so $\mathbb{E}\left|\mathbb{I}_X(\theta_n) - f(\theta_n)\mathbb{I}_\varepsilon(\theta_n)\right| = O\left(\sqrt{a_n}\right)$. We prove now $i)$ of (18). We suppose first that $\theta \in]0,\pi[$. The spectral representation of X_t yields:

$$\mathbb{E}\mathbb{I}_X(\theta_n) = \frac{1}{n}\int_{-\pi}^{\pi}\left|\Delta_n(x-\theta_n)\right|^2 f(x)\,dx$$

Hence $\mathbb{E}\mathbb{I}_X(\theta_n) - f(\theta_n) = \frac{1}{n}\int_{-\pi}^{\pi}\left|\Delta_n(x-\theta_n)\right|^2\left[f(x) - f(\theta_n)\right]dx$.

As in Robinson (1995a), we bound this integral by splitting $[-\pi,\pi]$ into several sub-intervals. Let us take $n \geq n_0$ such as $|\lambda_n| < \varepsilon$ and $\left[\theta + \frac{\lambda_n}{2}, \theta + 2\lambda_n\right] \subset V(\theta)$.

$$\begin{aligned} \left|\int_{-\pi}^{\theta-\varepsilon} + \int_{\theta-\varepsilon}^{\pi}\right| &\leq \frac{1}{n}\max_{|u|>\varepsilon}\left|\Delta_n(u)\right|^2\int_{-\pi}^{\pi}\left|f(x) - f(\theta_n)\right|dx \\ &\leq \frac{1}{n(x-\theta_n)^2}\left[\int_{-\pi}^{\pi}|f(x)|\,dx + 2\pi f(\theta_n)\right] \leq \frac{C}{n\varepsilon^2} = O\left(n^{-1}\lambda_n^{-1}\right) \end{aligned}$$

f is integrable and $f(\theta_n)$ is bounded by the continuity of f at θ. The same argument yields:

$$\begin{aligned}\left|\int_{\theta-\varepsilon}^{\theta+\frac{\lambda_n}{2}}\right| &\leq \frac{C}{n}\int_{\theta-\varepsilon}^{\theta+\frac{\lambda_n}{2}}\left|\Delta_n(x-\theta_n)\right|^2 dx\times\underbrace{\max_{x\in\left[\theta-\varepsilon,\theta+\frac{\lambda_n}{2}\right]}\left|f(x)-f(\theta_n)\right|^2}_{=O(1)}\\ &\leq \frac{C}{n}\int_{-\lambda_n-\varepsilon}^{-\frac{\lambda_n}{2}}\left|\Delta_n(u)\right|^2 du \leq \frac{C}{n}\int_{\frac{\lambda_n}{2}}^{\pi}\frac{du}{u^4}\\ &= O\left(\frac{1}{n\lambda_n}\right)\end{aligned}$$

Similarly, $\left|\int_{\theta+2\lambda_n}^{\theta+\varepsilon}\right| = O\left(\frac{1}{n\lambda_n}\right)$. Now, let $\mathbb{I}_n = \int_{\theta+\frac{\lambda_n}{2}}^{\theta+2\lambda_n}$.

$$n\left|\mathbb{I}_n\right| \leq \sup_{x\in\left[\theta+\frac{\lambda_n}{2},\theta+2\lambda_n\right]}\left|f'(x)\right|\int_{\theta+\frac{\lambda_n}{2}}^{\theta+2\lambda_n}\left|x-\theta_n\right|\left|\Delta_n(x-\theta_n)\right|^2 dx$$

From $\mathbf{H}_3$, $n\left|\mathbb{I}_n\right| \leq \left(\frac{\lambda_n}{2}\right)^{-d}\int_{-\frac{\lambda_n}{2}}^{2\lambda_n}|u|\left|\Delta_n(u)\right|^2 du \leq \left(\frac{\lambda_n}{2}\right)^{-d}\int_{-\frac{\lambda_n}{2}}^{2\lambda_n}|u|\left|\Delta_n(u)\right|^2 du$

$\leq 2\left(\frac{\lambda_n}{2}\right)^{-d}\int_0^{2\lambda_n}|u|\left|\Delta_n(u)\right|^2 du \leq 4\left(\frac{\lambda_n}{2}\right)^{-d}\int_0^{2\lambda_n}\left|\Delta_n(u)\right| du$ where we use (2).

But, classically:

$$\begin{aligned}\int_0^{2\lambda_n}\left|\Delta_n(u)\right| dx &= 2\int_0^{\lambda_n}\left|\frac{\sin(nu)}{\sin(u)}\right| du = 2\int_0^{\lambda_n}\left|\frac{\sin(nu)}{u}\right| du + O(1)\\ &= 2\int_1^{n\lambda_n}\left|\frac{\sin(x)}{x}\right| dx + O(1) \leq 2\int_1^{n\lambda_n}\frac{dx}{x} + O(1) = O\left(\log(n\lambda_n)\right)\end{aligned}$$

So $\mathbb{I}_n = O\left(n^{-1}\lambda_n^{-d}\log(n\lambda_n)\right) = O\left(n^{-1}\lambda_n^{-1}\log(n\lambda_n)\right)$ because $d \leq 1$.

Finally $\mathbb{E}\mathbb{I}_X(\theta_n) - f(\theta_n) = O\left(\frac{1+\log(n\lambda_n)}{n\lambda_n}\right)$.

For point $ii)$ of (18) we know[2] that $\mathbb{E}\left(\mathbb{J}_X(\theta_n)\overline{\mathbb{J}_\varepsilon(\theta_n)}\right) = \frac{\sigma^2}{2\pi n}\int_{-\pi}^{\pi}\left|\Delta_n(x-\theta_n)\right|^2\Psi(x)\,dx$ and the proof is identical.

[2]Starting from the stationary vector $\widetilde{X_t} = [X_t, \varepsilon_t]'$, we write its spectral density matrix $\widetilde{f(\omega)} = \begin{bmatrix} f(\omega) & \frac{\sigma^2}{2\pi}\Psi(\omega) \\ \frac{\sigma^2}{2\pi}\overline{\Psi(\omega)} & \frac{\sigma^2}{2\pi}\end{bmatrix}$. From the spectral representation of $\widetilde{X_t}$ we get easily : $J_X(\theta_n) = \int_{-\pi}^{\pi}\Delta_n(x-\theta_n)\,dZ_X(x)$ and $J_\varepsilon(\theta_n) = \int_{-\pi}^{\pi}\Delta_n(x-\theta_n)\,dZ_\varepsilon(x)$ then :

The case $\theta = \pi$ is almost identical. This time, $\lambda_n < 0$ and we write: $\left|\int_{-\pi}^{\pi-\varepsilon}\right| + \left|\int_{\pi-\varepsilon}^{\pi+2\lambda_n}\right| = O\left(\frac{1}{n|\lambda_n|}\right)$. Lastly, $\mathbb{I}_n = \int_{\pi+2\lambda_n}^{\pi}$ satisfies:

$$\begin{aligned} n\left|\mathbb{I}_n\right| &\leq \sup_{x\in[\pi+2\lambda_n,\pi]}\left|f^{'}(x)\right| \int_{\pi+2\lambda_n}^{\pi} |x-\theta_n|\left|\Delta_n(x-\theta_n)\right|^2 dx \\ &\leq \left(\frac{|\lambda_n|}{2}\right)^{-d} \int_{\lambda_n}^{-\lambda_n} |u|\left|\Delta_n(u)\right|^2 du \end{aligned}$$

We conclude then as before.

■

$E\left(J_X(\theta_n)\overline{J_\varepsilon(\theta_n)}\right) = \frac{1}{n}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\Delta_n(x-\theta_n)\overline{\Delta_n(y-\theta_n)}E\left(dZ_X(x)\overline{dZ_\varepsilon(y)}\right)$

But $E\left(dZ_X(x)\overline{dZ_\varepsilon(y)}\right) = 0$ if $x \neq y$, and $\widetilde{f_{X\varepsilon}(x)} = \frac{\sigma^2}{2\pi}\Psi(x)$ otherwise.

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